Nuclear Levels of F^{20}†

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The nuclear levels of F^{20} have been investigated at neutron energies from about 1 to 300 keV. Only a few resonances can be attributed to *s*-wave levels, the remaining ones that can be analyzed being attributable to *p*-wave levels. The two resonances near 27 and 49 keV were investigated extensively and can be attributed to *p*-wave levels. However, the former assumption of mutual interference between this pair of levels was ruled out because their spins of 2 and 1, respectively, appear to be well determined and each resonance exhibits an asymmetrical shape which is just the reverse of what it should be for mutual interference. The cluster of resonances near 100 keV appears to consist of three *p*-wave levels (one each with $J=0$, 1, and 2) and a fourth resonance attributable to an *s*-wave level. Two other small peaks occur on the high-energy side of the cluster. For the levels that could be analyzed, the *s*-wave strength function was found to be small and the *p*-wave one very large. The measurements were made by the method used previously, but the analysis included some recently developed methods.

I. INTRODUCTION

THIS paper is concerned with a study of the unbound nuclear energy levels of F^{20} in the keV region by the $F^{19}(n,n)$ process. The motivation, as in the previous studies[1] of the levels of Na^{24} and Al^{28}, is to determine (1) the number and type of levels, (2) the parameters of the levels, (3) the strength functions, (4) the distribution of the levels, and (5) whether or not the levels of F^{20} exhibit a pattern similar to the other nearby odd-odd nuclei.

Initial measurements on fluorine were made by Bockelman[2] some time ago with large neutron energy spreads. He observed eight peaks below 700 keV. Later Wills *et al.*,[3] working with a 20-keV neutron energy spread, revealed a number of broad peaks for F^{20} in the region from about 0.5 to 5.0 MeV. Irregularities around these peaks suggest that they might consist of clusters of narrow levels.

The resonance levels up to 160 keV were better resolved in a more intensive study by Hibdon and Langsdorf,[4] who used a small neutron energy spread. Measurements up to about 300 keV, made later at Duke University,[5] confirmed the results of Hibdon and Langsdorf within the energy range common to both studies. With the narrower neutron energy spreads now in use, however, known levels can be better resolved and an intensive search might reveal levels too narrow to be found in the earlier work. The present paper covers the resonances in F^{20} up to about 300 keV. The energies quoted are the laboratory-system values for the incident neutrons, unless otherwise noted.

II. EXPERIMENTAL METHOD

The measurements were made by the method described in connection with earlier experiments.[1,6] In brief, the present measurements were made by use of neutrons emitted at an angle of 120° with respect to the direction of the proton beam. The proton beam from the Argonne 5-MeV Van de Graaff accelerator is first mass analyzed by a 17° deflection in a magnetic field and then passes through a 90°-electrostatic analyzer having a radius of 40 in. Entrance and exit slits to the electrostatic analyzer were adjusted to widths of 30 and 40 mils, respectively. Lithium targets having stopping powers from 0.60 to 0.80 keV are used as the neutron source. A new target is evaporated daily from lithium believed to be 99.9% Li^7 and a yield curve is run. The threshold energy is determined by extrapolating the straight portion of the yield curve to zero. This locates the front of the lithium target and the energy is taken to be 1.881 MeV. When used for neutron energy calibration for measurements at 120°, this point differs from the mean neutron energy by roughly one-fourth the target thickness. Therefore, the neutron energy has always been corrected to mean neutron energy by subtracting one-fourth the target thickness from the energy setting. The neutron counter consists of fifty BF_3 proportional counters (with the boron enriched to 96% B^{10}) arranged in a cylinder of paraffin encased in a large shield with a collimator penetrating the entire length of the shield. A "detector sample" of graphite or lucite (6 in. thick) is placed in the collimator near the midpoint of the counters to scatter neutrons into the sensitive region. The detector sample is removed to obtain the background. The defining slit of the collimator is 13.5 in. from the source of neutrons. This slit is rectangular in shape with a height of 1.25 in. and ordinarily a width of $\frac{1}{4}$ in. However, for studies near the

† Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] C. T. Hibdon, Phys. Rev. **114**, 179 (1959); **118**, 514 (1960); **122**, 1235 (1961); **124**, 500 (1961). For a description of the apparatus see C. T. Hibdon, *ibid.* **108**, 414 (1957).

[2] C. K. Bockelman, Phys. Rev. **80**, 1011 (1950).

[3] J. E. Wills, Jr., J. K. Bair, H. O. Cohn, and H. B. Willard, Phys. Rev. **109**, 891 (1958).

[4] C. T. Hibdon and A. Langsdorf, Jr., Phys. Rev. **98**, 223 (1955), Sec. A.

[5] P. W. Crutchfield, W. Haeberli, and H. W. Newson, Bull. Am. Phys. Soc. **2**, 33 (1957); E. Merzbacher, H. W. Newson, and P. W. Crutchfield, *ibid.* **2**, 33 (1957). See also H. W. Newson, E. G. Bilpuch, F. P. Karriker, L. W. Weston, J. R. Patterson, and C. D. Bowman, Ann. Phys. (N. Y.) **14**, 365 (1961).

[6] C. T. Hibdon, Nucl. Instr. Methods **17**, 177 (1962).

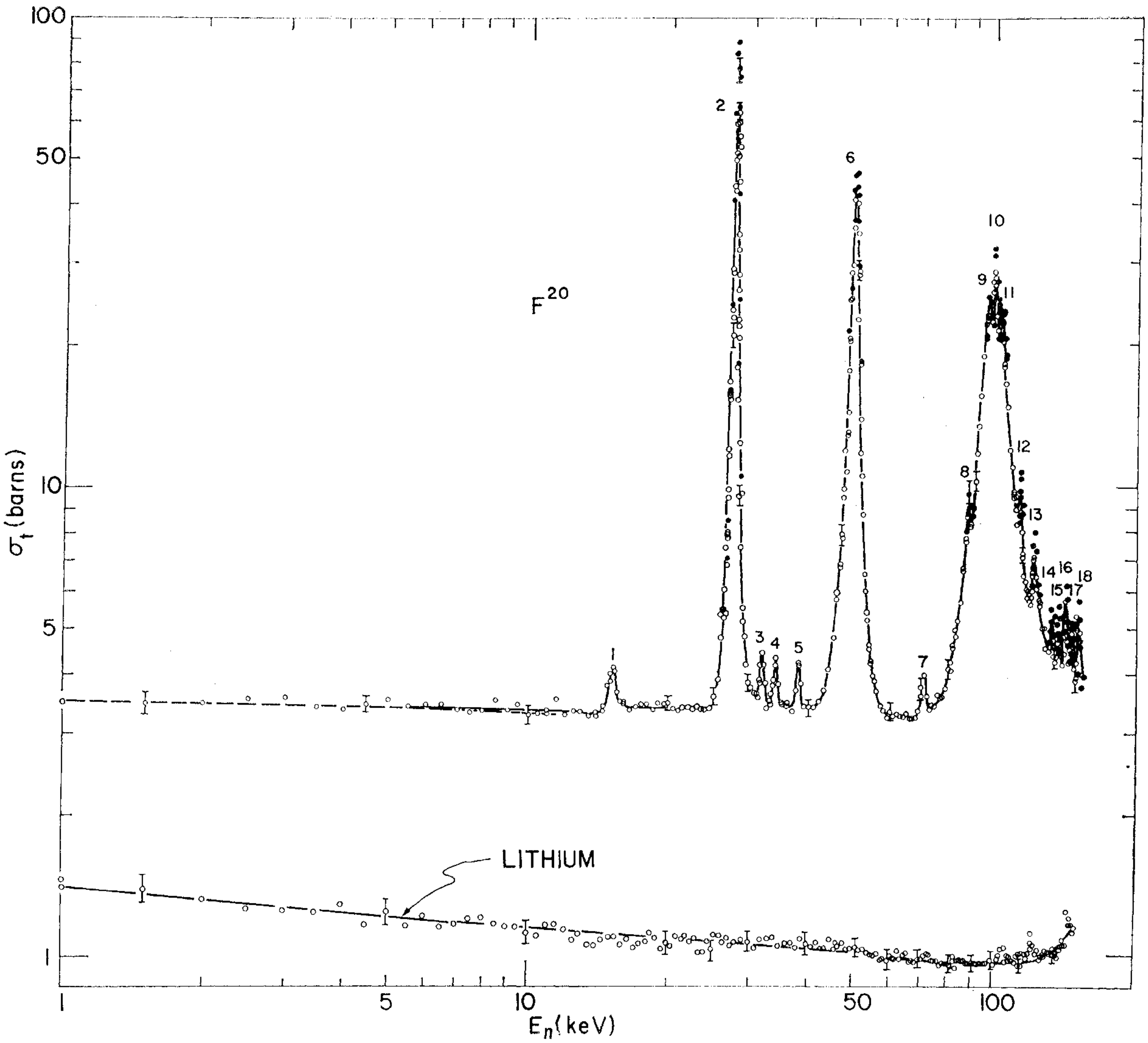

FIG. 1. Neutron total cross section of lithium and fluorine from about 1 to 150 keV. Open circles show data obtained by flat-detection; solid circles data by self-detection. The data for fluorine were obtained by subtracting the data for lithium from the data for LiF samples.

peaks of narrow resonances and in some narrow valleys between closely spaced peaks, a width of $\frac{1}{8}$ in. is used.

Lithium fluoride crystals grown by the Harshaw Chemical Company and polished by the optical shop at Argonne were used for transmission samples. These samples are 2.5 in. in diameter and range in thickness from 50 mils to 1.0 in. Because natural lithium was used to grow the crystals, it was necessary to determine the cross section of LiF and lithium at each setting. The cross section of LiF was taken to be the sum of the two elements.

High-purity samples of lithium metal were prepared and sealed in aluminum cylinders in a way similar to that for the sodium samples[1,6] used in previous measurements. Since molten lithium metal alloys with aluminum, the lithium metal was first poured into thin-walled copper tubing, allowed to cool, pressed, removed from the copper tubing and canned in an aluminum cylinder with end windows of 1-mil stainless steel. This process was carried out in a dry box flooded with argon. The lithium samples range in thickness from 0.2 to 3.0 in. Because of the wide lithium resonance near 250 keV, obtaining the cross section of F from that of LiF involved a large subtraction in this energy region. To avoid this large correction, the region near the lithium resonance was reinvestigated by use of Teflon samples, but no significant difference was observed in the resulting cross section of fluorine. All samples were arranged in a sample changer so that any desired thickness could be obtained immediately to provide a transmission around 50 to 60%. The presence of the wheels that carry the samples does not alter the background by a detectable amount.

Two types of transmission measurements were made: (1) flat detection, in which the response is nearly independent of neutron energy because of the use of a nonresonant "detector sample" (usually a graphite plug 6 in. long placed in the collimator channel[1,6] to scatter the neutrons into the counters), and (2) self-detection, which discriminates against off-resonance neutrons by using a resonant detector sample made of the same material as the transmission sample, or at least a material containing the nuclide under study. Hence, the peak cross section determined by this method comes closer to the true peak height than does that determined by flat detection. Moreover, narrow valleys between closely spaced resonances can be better resolved by self-detection. Thus, the parameters of the resonances can be more accurately determined by a combination of the two methods of measurement than by flat detection alone. However, the widths of some resonances (such as Nos. 1, 3, 4, etc., in Fig. 1) are so small that one cannot resolve them sufficiently by either method of measurement to determine their parameters.

The data shown in the various figures are the raw data with only two corrections applied: (1) At each energy setting, the data obtained were normalized by comparison with the count recorded by a BF_3 long counter placed 50 in. from the neutron source at an angle of 0° with respect to the direction of the proton beam. (2) Then the data were corrected for background, which was obtained by removing the scattering sample from the counting equipment.[1,6]

The large difference between the peak heights observed by flat detection and self-detection for the 27-keV resonance is a clear indication that the width of this level is less than the neutron energy spread. Hence, this resonance provides a means for obtaining an estimate of the neutron energy spread. The observed width of this resonance at half its maximum height above the surrounding background is close to 400 eV. This width is interpreted to be the over-all effective neutron energy spread and is approximately half the thickness of the lithium target determined from the yield curve.

III. EXPERIMENTAL RESULTS

The neutron cross-section data obtained are shown in Figs. 1, 2, 3, 4, and 5. The general features up to

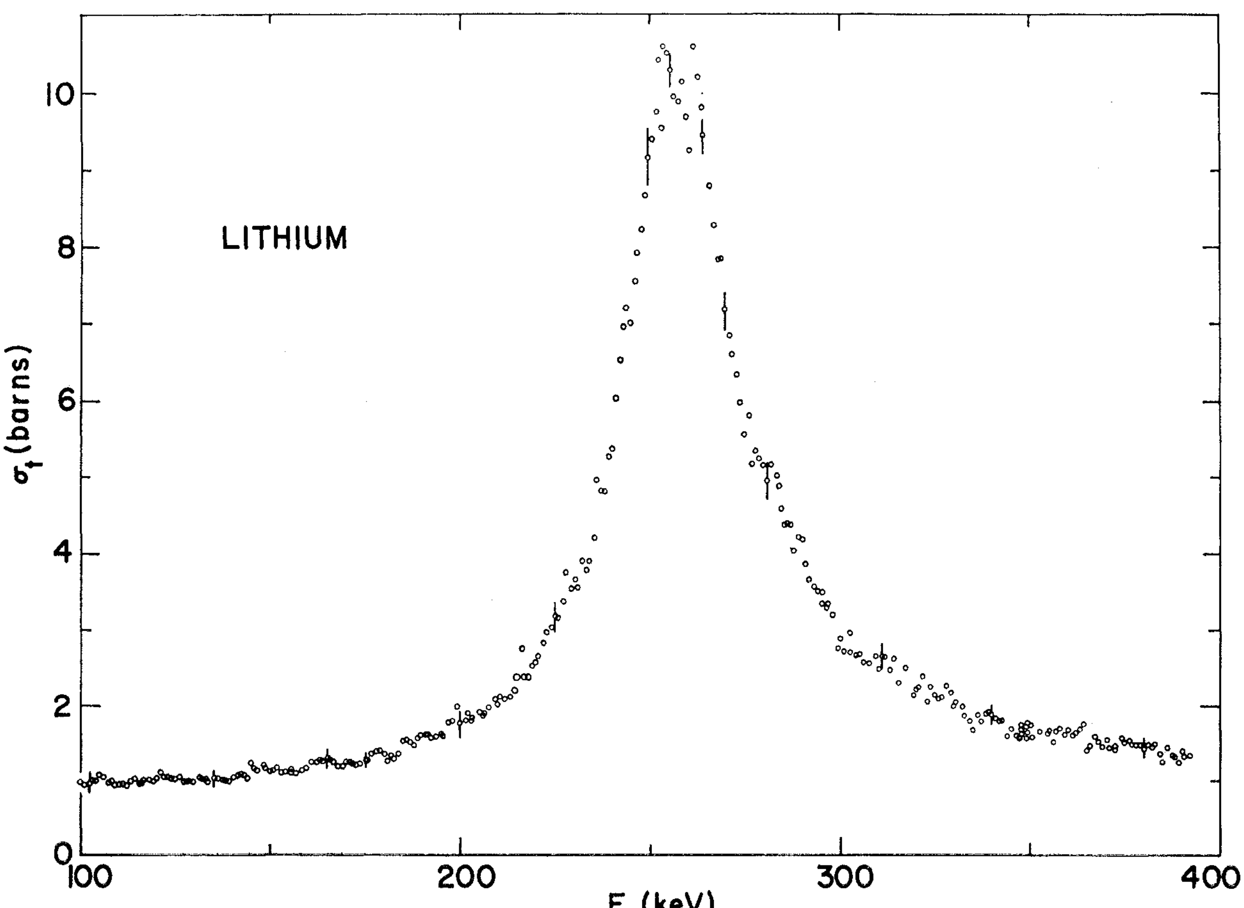

FIG. 2. Neutron total cross section of lithium from 100 to 390 keV.

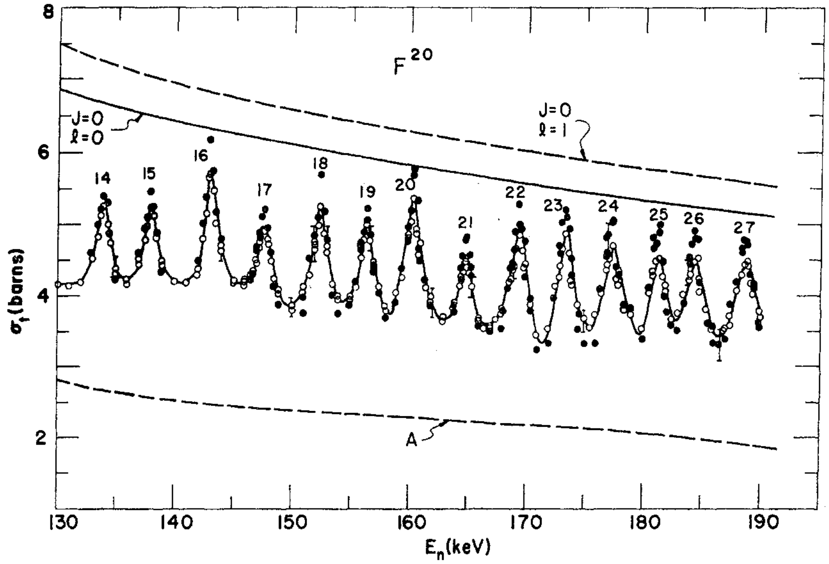

FIG. 3. Neutron total cross section of fluorine from 130 to 190 keV. The curve shown by a solid line depicts the trend of the data obtained by flat detection. Solid circles represent the data obtained by self-detection. The loci of the theoretical single-level peak heights for $J=0$, $l=0$, 1 include only elastic neutron scattering. Curve A is composed of the potential scattering of the wings of s-wave levels.

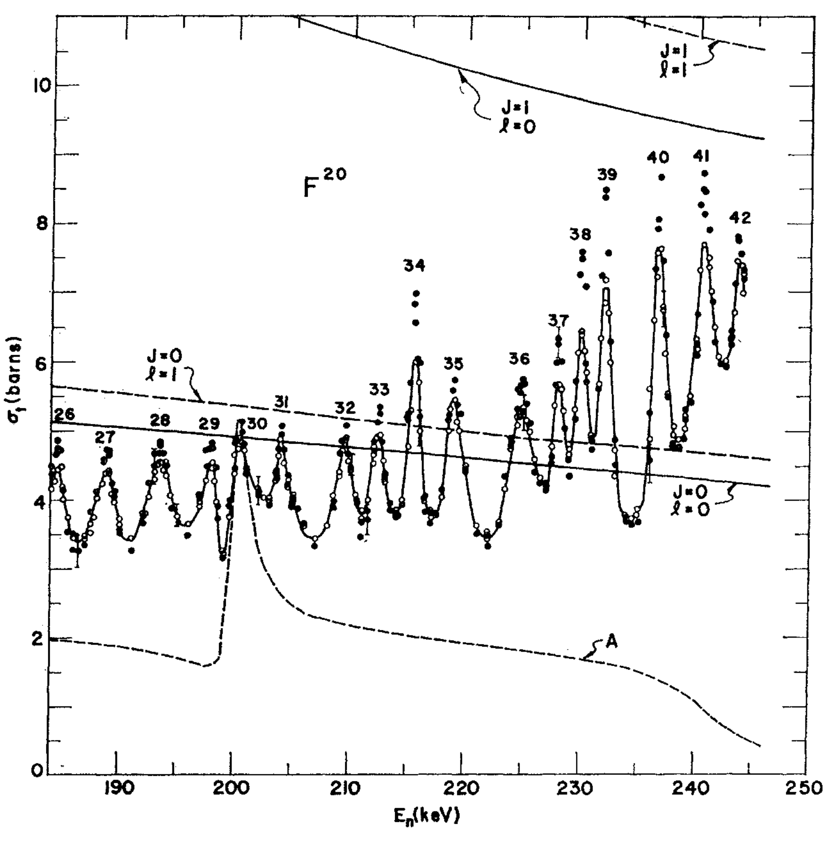

FIG. 4. Neutron total cross section of fluorine from 184 to 244 keV. The solid curve follows the trend of the flat-detection data. Data obtained by self-detection are represented by solid circles. The loci of the theoretical single-level peak heights for $J=0$ and 1 include only elastic neutron scattering. Curve A is the combined contributions of the s-wave levels and the potential scattering.

about 150 keV are displayed in a semilogarithmic plot in Fig. 1. The results obtained for lithium in this range are also plotted in Fig. 1 because the data were taken concurrently with those for LiF and were used in obtaining the plot for fluorine. Figure 2 shows the remainder of the data for lithium at higher energies and includes the large peak near 250 keV. Peak Nos. 2 and 6, and a broad peak near 100 keV were known previously.[7] Some small peaks (Nos. 1, 3, 4, 5, and 7) were first observed in the present measurements. Peak No. 1 at

[7] *Neutron Cross Sections*, compiled by D. J. Hughes and R. B. Schwartz, Brookhaven National Laboratory Report BNL-325 and Suppl. No. 1, 1960 (U. S. Government Printing Office, Washington, D. C., 1958), 2nd ed.

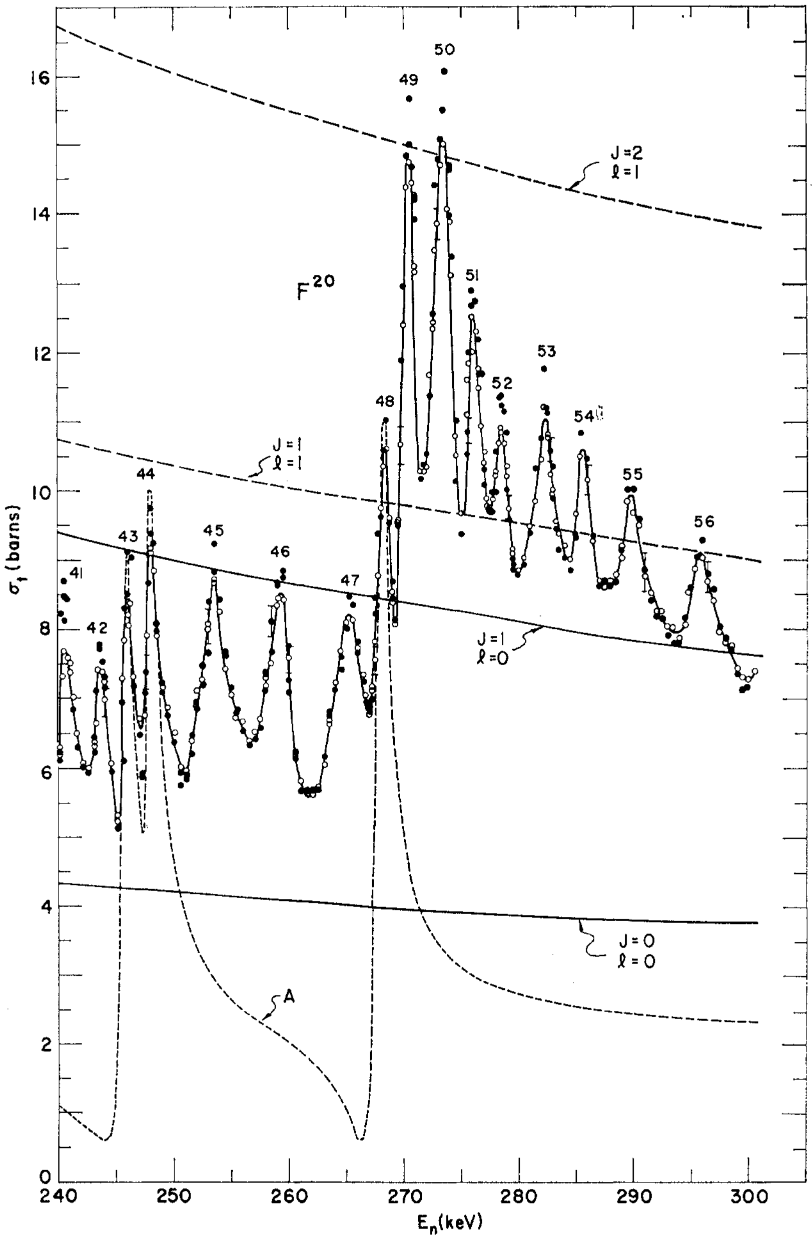

FIG. 5. Neutron transmission cross section of fluorine from 240 to 300 keV. The solid curve indicates the trend of the flat-detection data. Points shown by solid circles were obtained by self-detection. The loci of the theoretical single-level peak heights for $J=0$, 1, 2 include only elastic neutron scattering. Curve A is composed of the wings of s-wave levels and the potential scattering.

15.25 keV was at first suspected to be due to an unknown chemical impurity in the LiF crystals or to erroneous measurements. The measurements were therefore repeated but the peak persisted. In order to rule out an impurity in the sample, this peak was studied still further by use of Teflon samples of known high purity. Some time later it was learned that Gibbons, Macklin, Miller, and Neiler[8] at Oak Ridge had also observed this peak by neutron absorption measurements. Comparison with the peak height of No. 2, the width of which is about 0.30 keV, suggests that the true width of peak No. 1 is of the order of 10 eV. Small peaks Nos. 3, 4, and 5 were again observed on repeating the measurements in the region between peak Nos. 2 and 6. This no doubt explains the fact that some high points were observed in this region in all previous experiments, and that the cross section remained higher than might be expected in the large valley between peak Nos. 2 and 6. Apparently these small peaks are real and have widths comparable to that of No. 1.

Figure 1 shows peaks Nos. 2 and 6 to be quite well resolved except near the peak of No. 2. Both flat- and self-detection measurements gave higher peak values than in former measurements. The wings of these

[8] J. H. Gibbons, R. L. Macklin, P. D. Miller, and J. H. Neiler (private communication).

resonances appear to be well resolved and hence their shapes are well revealed. The known broad peak near 100 keV has been investigated at this laboratory in the course of several previous experiments with larger neutron energy spreads than are now in use. On each occasion there were indications that this broad peak is composed of a number of resonances. The present investigation revealed a cluster of narrow levels. All of these peaks are well enough resolved to be seen in Fig. 1. However, in a later plot (Fig. 9) on an expanded scale, they are more clearly distinguishable.

Up to 130 keV, no peak shown has the clearly resolved minimum on its low-energy side nor the characteristic asymmetrical shape to identify it as an *s*-wave level. However, some features of the data, to be discussed later, appear to defy explanation unless one of the peaks in the large cluster near 100 keV can be attributed to an *s*-wave level.

Figures 3, 4, and 5 show the remainder of the data for F^{20} up to 300 keV. The data in Figs. 3 and 4 show a large number of small peaks up to about 225 keV with an indication of only one *s*-wave level, No. 30. Above 225 keV, many more narrow peaks were observed. They exhibit much higher peak cross sections than the resonances between 150 and 225 keV. Peaks Nos. 43, 44, and 48 in Fig. 5 appear to be *s*-wave levels. The general features of all of the data up to 300 keV certainly indicate a preponderance of levels having angular momentum $l>0$.

IV. IDENTIFICATION OF THE LEVELS OF F^{20}

A. Preliminary Topics

Some value for the potential scattering cross section σ_p must be used in order to fit the wings of the resonances. Usually a reasonably accurate value of σ_p can be determined by one or both of two methods: (1) The minimum on the low-energy side of an isolated well-resolved *s*-wave level and a knowledge of the spin J of the level yields the value of σ_p. Unfortunately, no isolated *s*-wave levels occur in the present measurements. (2) The other possibility is the value of the cross section in a region completely free of the wings of levels. The only region in fluorine that appears to be free of the wings of levels is below 15 keV. Here the cross section averages about 3.4 b and this might be a possible value for σ_p. However, a number of compelling reasons cast doubt on this value for σ_p. (1) This value is larger than would be expected for a light nucleus. (2) In the region near 65 keV, the cross section is about 3.2 b and in addition to σ_p consists of contributions from the wings of nearby levels. (3) Data at high energies[3,7] appear to be inconsistent with a large value of σ_p. (4) The large density of unbound levels is expected to be a continuation of a large density of bound levels; hence a sizable contribution from the wings of the bound levels undoubtedly extends into the region of the unbound levels. It, thus, appears that the true value of σ_p is considerably less than 3.4 b but the correct value cannot be determined directly from the data. Hence, the alternative is to assume a reasonable value for σ_p and attempt to determine the contributions from the wings of levels. The value used is $\sigma_p=4\pi\lambda\!\!\!^{-2}\sum_l(2l+1)\sin^2\delta_l$, where the δ_l are the hard-sphere phase shifts: $\delta_0=R/\lambda\!\!\!^{-}$ etc. The value σ_p given by this expression agrees with the data[1] for Na^{24} and Al^{28}. It is, therefore, assumed throughout this paper. Then a value of the nuclear radius $R=(0.14\times10^{-12})A^{1/3}$ cm yields about 1.82 b for σ_p in the present energy region.

Fluorine has only one known stable isotope, F^{19}. Hence, all observed neutron resonances correspond to levels of the compound nucleus F^{20}. The nuclear spin of F^{19} is $I=\frac{1}{2}$ and the parity of the ground state is even.[9] For *s*-wave ($l=0$) neutron interactions, the levels formed have $J=0$ or 1 and even parity; *p*-wave levels ($l=1$) have $J=0$, 1, or 2 and odd parity; *d*-wave ($l=2$) levels have $J=1$, 2, or 3 and even parity; and *f*-wave ($l=3$) levels have $J=2$, 3, or 4 and odd parity. In the event any *g*-wave ($l=4$) levels are excited, they have $J=3$, 4, or 5 and even parity. Thus for $l\leqslant4$ it is possible to excite any of 11 types of levels differing in J or parity. Previous experience with the nuclei[1] Na^{24} and Al^{28} indicated that one might expect to excite any or all of these levels and even levels having higher values of J and l. It should be remembered that for F^{20} only three values of l are possible for each value of J and only $l=0$ and 1 are possible for $J=0$.

In the presence of other competing processes, such as (n,γ), (n,p), (n,α), or inelastic scattering, the interaction of the neutrons with the nuclei may not be predominantly elastic scattering. In such cases only lower limits can be assigned to the values of J and therefore the level widths determined will exceed the true widths unless corrections for other than elastic scattering can be made. Since the threshold[7,10] for the (n,p) and (n,α) reactions are well above 3 MeV, these two processes are unambiguously ruled out for the range covered in this investigation. Early measurements of the (n,γ) reaction by Henkel and Barschall[11] showed a small peak near 265 keV and another near 590 keV. Recent measurements by Gabbard, Davis, and Bonner[12] show peaks at 27, 48, 100, 177, 270, 308, 388, 425, 500, and 600 keV, with smaller peaks at higher energies. The largest peak height observed, even when corrected for neutron energy spread, was 300 mb at 27 keV. Hence, it appears that the (n,γ) process makes only small contributions in the present region. However, some

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[9] *Energy Levels of Light Nuclei VI*, compiled by F. Ajzenberg-Selove and T. Lauritsen (North-Holland Publishing Company, Amsterdam, 1959), reprinted from Nucl. Phys. **11**, 1 (1959).

[10] J. B. Marion and R. M. Brugger, Phys. Rev. **100**, 69 (1955); N. A. Bostrom, E. L. Hudspeth, and I. L. Morgan, *ibid.* **99**, 643 (1955), Sec. A.

[11] R. L. Henkel and H. H. Barschall, Phys. Rev. **80**, 145 (1950).

[12] F. Gabbard, R. H. Davis, and T. W. Bonner, Phys. Rev. **114**, 201 (1959).

specific cases will be explored later. Inelastic scattering has been studied by Freeman,[13] but below about 120 keV there appears to be little if any contribution to interfere with the analysis. Above 120 keV, inelastic scattering begins and increases with energy over a considerable region. This process is not negligible, so no analyses of the levels appear to be feasible in this region unless the inelastic scattering can be taken into account.

The method of analysis used can be described briefly. Provisional parameters of the levels were employed to obtain single- and multiple-level plots which are compared with the data. These parameters are repeatedly revised until the plots show the best obtainable agreement with the data and account for the data except near the peaks of narrow levels. The analyses could be simplified if the parameters of s wave and all other wide levels could be determined and the extensive wings of these levels subtracted from the data. Because of the overlapping of wings of some levels and the strong mutual interference of other levels, no level could be isolated. Hence, all the levels of this group were analyzed simultaneously. Although each level or group of levels is discussed separately later, the wings of all overlapping or interfering levels are included in each plot. The equations used to obtain the plots were given in a previous publication.[14] Equation (2) on page 182 of Ref. 14 was used for multiple-level plots and Eq. (3) for single-level plots. The many tedious calculations needed were performed by the Applied Mathematics Division with the IBM-704 computer. This program includes the variation of the widths of the levels with neutron energy E_n in accordance with the relation $\Gamma_n=2P_l\gamma_l^2$, where $P_0=x$, $P_1=x^3/(x^2+1)$, etc. with $x=R/\lambda\!\!\!^{-}$. By use of this relation for Γ_n rather than a constant value for each level, the wings of the levels can be extended as far as needed. In addition to the method just outlined, some recently developed methods to be described as needed were also applied to most of the resonances.

B. Cross Section Below 15 keV

As noted before, the cross section below 15 keV appears to consist of the potential scattering plus contributions from the wings of levels. Most of these levels must be bound levels which have not yet been resolved.[9] Hence, their locations and contributions must be inferred from the present data. The following possibilities were considered. Bound p-wave levels can contribute very little to the cross section below 15 keV because of the small penetrability factor P_1 in this region. A single s-wave level can hardly maintain a flat cross section over this region unless it were quite wide and relatively far removed. It was found that two bound s-wave levels assumed to be located at -65 and -30 keV with reduced widths γ_0^2 of 20 and 7.5 keV, respectively, can produce a reasonable fit to the data; but three bound levels at -80, -60, and -30 keV with reduced widths of 11, 11, and 9 MeV, respectively, can maintain a flat cross section to higher energies in the region of unbound levels. The curve shown by a broken line up to about 15 keV in Fig. 1 represents the multiple-level contribution from the s-wave levels, peaks Nos. 11, 43, 44, 48, and the three bound levels plus $\sigma_p=1.82$ b. This curve extends throughout the region covered and is used in the analyses of all resonances. However, the assumed bound levels and their parameters are not unique. Apparently a group of s-wave bound levels must contribute a sizable amount to the cross section in the region of unbound levels and this contribution extends to rather high energies. It is to be noted that in F^{20} the bound levels to which probable spins and parities have been assigned tend to occur in groups.[9]

C. Analysis of the 27-keV Resonance

This well-known resonance is a relatively narrow peak which at first sight appears to be well isolated from other peaks. Therefore, its width, spin, and parity have received considerable study. The present data near this peak are displayed on a semilogarithmic plot in Fig. 6. Initial measurements were made with a lithium target with a stopping power of about 0.75 keV (determined from the yield curve).[1] The entrance slit to the collimator of the neutron counter was $\frac{1}{4}$ in. wide. A peak cross section of 53 b was observed by flat detection with a transmission sample 100 mil thick (1.51×10^{22} molecules/cm^2); by self-detection it was 74 b with a 50-mil sample (0.756×10^{22} molecules/cm^2). For flat detection, a graphite detector sample 6 in. thick was used; for self-detection near the peak, a Teflon detector sample[6] 125 mils thick. Later, the region from about 26.5 to 28.0 keV was restudied by use of a lithium target of slightly less than 0.70-keV stopping power and a collimator entrance slit $\frac{1}{8}$ in. wide. By flat detection, a peak cross section of 64.7 b was observed and by self-detection 88.8 b.

The $F^{19}(n,\gamma)F^{20}$ measurements made by Gabbard *et al.*[12] yielded a peak height of about 0.3 b after correction for neutron energy spread. Moreover, for the present measurements near the peak of this resonance, a 300-mil Teflon detector sample yielded about 65% of the counting rate obtained with the 6-in. graphite sample and a 125-mil Teflon sample yielded 45%. This amount of scattering and the very small amount of neutron absorption observed is taken to mean that this peak is mainly a scattering level.

The sizable increase observed in the peak height when self-detection (with its higher resolution) replaces flat detection is a clear indication that the true peak height was not attained by either method and that the width of the level is comparable with or less than the neutron energy spread. Since the peak height observed by self-detection considerably exceeds the theoretical

[13] J. M. Freeman, Phys. Rev. 99, 1446 (1955).
[14] C. T. Hibdon, Phys. Rev. 114, 179 (1959), especially p. 182.

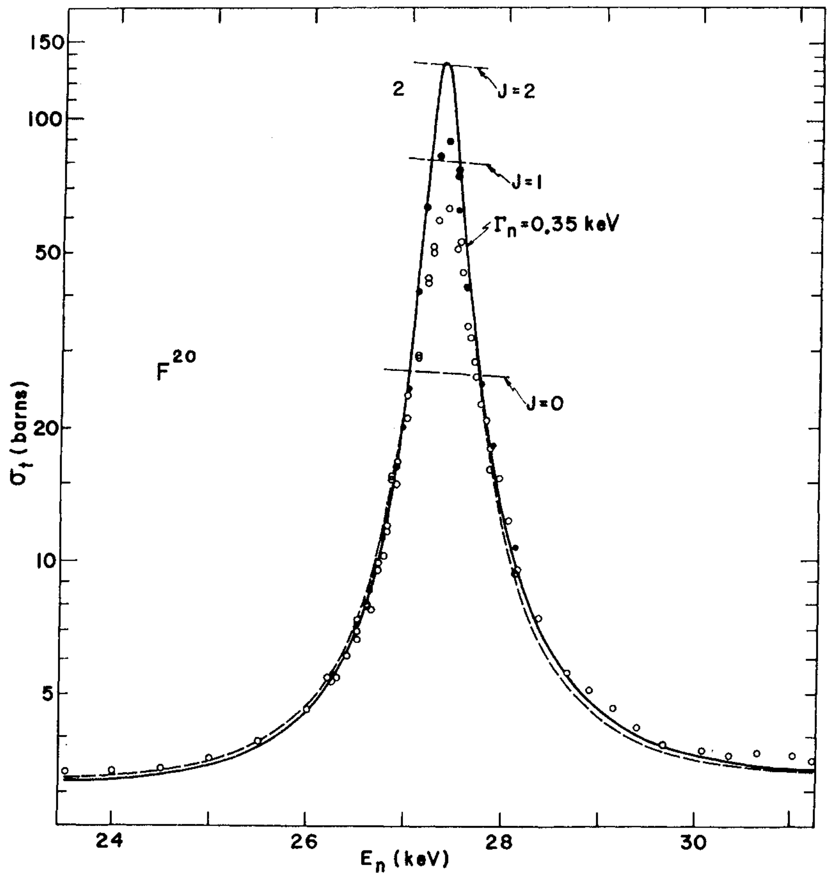

FIG. 6. Neutron total cross section of fluorine in the region of the 27-keV resonance. Points shown by open circles were obtained by flat detection. Solid circles show points obtained by self-detection. The solid curve for $\Gamma_n=0.35$ keV was obtained by a multiple-level plot of peaks Nos. 2 and 10 and an assumed bound level at -4.0 keV. The broken curve is a single-level plot for peak No. 2. Neither curve was corrected for neutron energy spread.

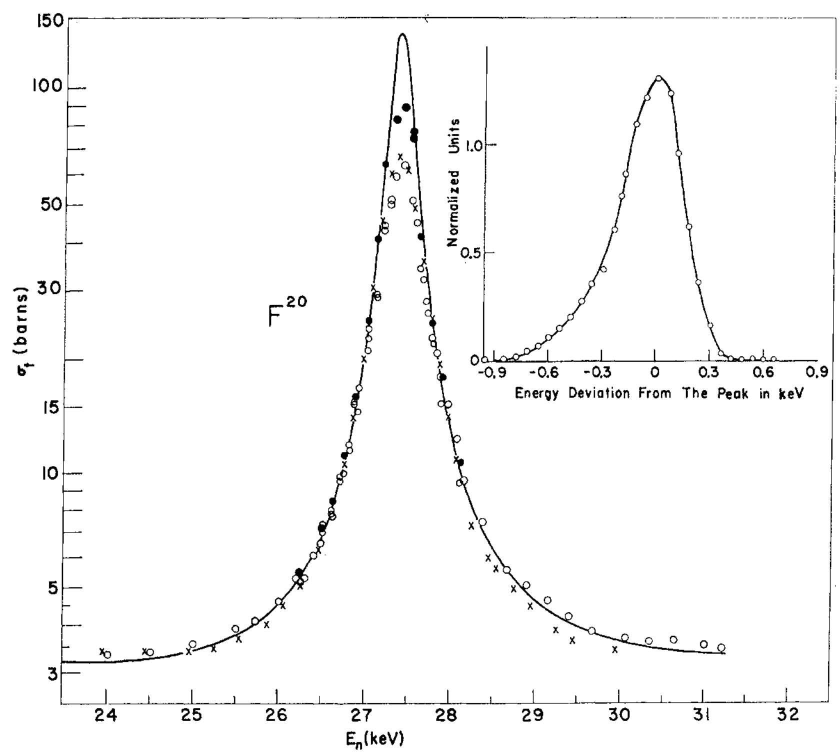

FIG. 7. Analysis of the 27-keV resonance of fluorine. Points shown by crosses were obtained by the method of Lane, Morchouse, and Phillips. These points represent the flat-detection data to be expected with a neutron distribution having an energy spread of 0.4 keV and the shape shown in the insert. The solid curve is a multiple-level curve obtained previously and is discussed in the text. The points shown by open circles were obtained by flat detection. Points shown by solid circles were obtained by self-detection.

value for $J=1$, the value of J appears to be 2. The present data and all previous data indicate that the two peak Nos. 2 and 6 in Fig. 1 are attributable to *p*-wave levels because no pronounced minima occur on their low-energy sides to identify them as *s*-wave levels and their widths seem to be quite large for *d*-wave levels in this energy region. Each peak is somewhat asymmetrical. However, the high cross section occurs in the high-energy wing of the 27-keV resonance and in the low-energy wing of the 49-keV resonance, just the reverse of what it would be if due to mutual interference. This lack of mutual intereference implies that the two peaks correspond to different values of J, as is also suggested by their peak heights. Since $J=1$ is much the most probable assignment for the 49-keV resonance, the value of J for the 27-keV resonance must be 2.

Because the 27-keV resonance is asymmetric, a single-level plot fails to follow the shape of its high-energy wing or fit its low-energy wing closely. Hence, a multiple-level plot with an assumed bound level was used. It agrees well with the shapes of the wings of this resonance. The curve which provides the best fit to the wings, shown in Fig. 6 by a solid line, was obtained for $\Gamma_n=0.35$ keV, $J=2$, and $l=1$. This curve includes the contributions from (1) a *p*-wave multiple-level plot for the two unbound levels shown by peaks Nos. 2 and 10 in Fig. 1, together with an assumed *p*-wave bound level with $E_n=-4.0$ keV and $\gamma_1^2=87.5$ keV, (2) the curve for the *s*-wave levels, and (3) a small contribution from the multiple-level plot for Nos. 6 and 9. The curve shown by a broken line in Fig. 6 is the corresponding single-level plot. These two plots indicate that the asymmetric shape of this level can be attributed in part to its natural single-level shape but to a greater degree to mutual interference with some bound level. These plots were obtained by use of the Breit-Wigner equations[14] unweighted for neutron resolution. When the neutron resolution is taken into account later, it will be seen that a width of about 0.30 keV is a better value. Undoubtedly, contributions from bound levels extend into this region and some part of these contributions is due to *p*-wave levels. The part coherent with the 27-keV resonance was localized to a single level at -4.0 keV, but this may be a fictitious level, which is equivalent to small contributions from many unresolved bound levels. An alternative is to assume a larger value of σ_p and less contribution from bound levels. However, it should be noted that the analysis of the 49.7-keV resonance, which is farther removed from the bound levels, agrees well with $\sigma_p=1.82$ b added to the multiple-level contribution from the *s*-wave levels and contributions of other nearby levels. The peak height and hence the spin of the 27-keV resonance have been studied further by a different method of analysis given by Lane, Morehouse, and Phillips.[15] They have developed a technique for determining the true resonance shape from a measured resonance shape when the instrumental resolution shape is known, or, conversely, for determining the resolution function if the true resonance shape is known. The neutron distribution which Lane *et al.* found in this way by use of the 2.08-MeV resonance in C^{12} was used in conjunction with their method in the analysis of the flat-detection data on the 27-keV resonance in F^{20}.

The program used by Lane *et al.* was revised by Mary Welsh and N. F. Morehouse of the Applied Mathematics Division and the calculations were performed by the IBM-704 computer. The results are given in Fig. 7, where the solid curve is the multiple-level plot obtained previously. The flat-detection data near the peak of the resonance could be best fitted by assuming a scattering peak height of about 128 b rather than the single-level height of 134 b. The (n,γ) process reduces the peak height by the factor Γ_n/Γ and apparently is sufficient to lower the elastic scattering peak by a few barns. The insert shows the neutron distribution used, normalized to a width of 0.4 keV at its maximum height. The crosses represent what one would expect to observe if a resonance with the shape shown by the solid curve were measured with a neutron distribution having an energy spread of 0.4 keV and the shape shown in the insert. The actual flat-detection data are shown by open circles, and the points obtained by self-detection are shown by solid circles. Hence, for an effective neutron energy spread of 0.4 keV, this method duplicates the flat-detection data except well out in the wings. It could not be expected to agree in the wings because a single-level expression was used in this method of analysis and it has already been shown that the shapes of the wings of this resonance are modified by interference with other resonances. This method does indicate that the spin of this level is $J=2$ and that the neutron energy spread is close to 0.4 keV. It was previously shown that the observed width of the 27-keV resonance indicated a value of 0.4 keV for the over-all neutron energy spread. The method of Lane *et al.*, however, provides no means by which both the flat- and self-detection data can be used with a correction for the neutron distribution to obtain the spin J. Therefore, a different analytical method has been sought.

Monahan and Hibdon[16] have developed a method of analysis to correlate the flat-detection and self-detection data in order to determine the parameters of those levels that have widths comparable to or less than the neutron energy spread. The method employs a single-level expression for the resonance shape and a combination of a rectangular and a Cauchy-neutron distribution. The two neutron distributions can be combined in different proportions to obtain the best fit to the data. The details of such a method are quite involved and are

[15] R. O. Lane, N. F. Morehouse, and D. L. Phillips, Nucl. Instr. Methods **9**, 87 (1960).

[16] J. E. Monahan and C. T. Hibdon (to be published).

TABLE I. Comparison of the observed cross section over the 27- and 49.7-keV resonances of fluorine with the values obtained by the method of analysis given by Monahan and Hibdon (Ref. 16). The quantity σ is the observed value of the cross section, corrected for the neutron scattering by the tantalum backing of the lithium target plus air scattering, and σ_c is the "expected measurement" defined in Sec. 4C. For the 27-keV peak, the parameters obtained were a neutron energy spread $\Delta E=0.35$ keV and $\Gamma_n=0.30$ keV; and the peak height corresponds to $J=2$ for $l=1$. For the 49.7-keV peak, $\Gamma_n=1.60$ keV and the peak height corresponds to $J=1$ for $l=1$.

	Flat-detection		Self-detection	
E_n (keV)	σ (b)	σ_c (b)	σ (b)	σ_c (b)
26.5	6.54	6.86	7.14	6.95
26.75	10.08	9.78	11.32	10.10
27.0	22.98	20.87	24.56	23.62
27.1	31.07	31.85	41.94	41.77
27.2	46.63	56.59	68.40	64.61
27.3	63.64	82.82	86.11	88.88
27.4	70.10	85.21	92.23	90.35
27.5	56.80	60.93	68.85	68.18
27.6	35.96	37.42	43.31	45.53
27.75	22.00	19.00	25.00	22.49
28.13	9.40	7.53	10.50	7.65
48.6	20.76	20.33	22.06	21.55
48.85	26.00	26.04	26.94	27.42
49.2	36.99	36.70	37.54	37.83
49.4	42.65	42.27	43.43	43.02
49.7	44.99	46.41	46.30	46.86
50.0	41.56	42.84	42.43	43.55
50.25	35.83	35.93	37.27	37.10
50.5	29.30	27.98	29.98	30.02
51.0	18.05	17.83	18.96	18.88

given in a separate paper.[16] This method gives the parameters of a level for various proportions of the rectangular and Cauchy neutron-energy distributions and an estimate of the experimental neutron-energy spread. By use of the parameters obtained and the experimental neutron-energy spread, points are computed over the region of the resonance for both flat-detection and self-detection. These computed points, referred to hereafter as the "expected measurements," represent the data one would expect to observe for the experimental neutron energy spread determined from the observed data. The 27-keV resonance of fluorine has been studied extensively by this method of analysis. The computed points are compared with the data in Table I and show a good over-all agreement. This method does indicate that the observed points obtained by flat-detection from 27.2 to 27.4 keV should be somewhat higher and that the high-energy wing is definitely more asymmetric in shape than can be accounted for by a single-level plot. Some of the low points in the low-energy wing may also be partly due to this asymmetry introduced by multiple-level interference. For $l=1$, the parameters given by this method are a peak height near the height corresponding to $J=2$, $E_r=27.35$ keV, $\Gamma_n=0.30$ keV, and an effective neutron-energy spread of 0.35 keV.

The tantalum backing for lithium targets may introduce a sufficient background scattering of off-resonance neutrons to reduce the observed peak heights of narrow resonances. Therefore, this scattering has been measured by both flat- and self-detection for the equipment used here.[6] This correction was applied to the narrow levels of F^{20} before making the analysis by the method of Monahan and Hibdon. The results obtained near the peaks of levels of various heights are shown in Table II. One sees that this correction, though usually small in comparison with the correction for neutron energy spread, is in the right direction to aid in determining the correct peak heights (and hence the spins) of resonances. For the small peaks, the correction is seen to be negligible. In the σ_1 column of Table II, the values given in parentheses are the "expected measurements" given by the method of Monahan and Hibdon.[16] The single-level peak heights are also given by the latter method and hence are much higher, being 128 b for the 27-keV resonance, 7.4 b for the 143-keV resonance, and 10.2 b for the 215.5-keV resonance, for example.

The width of 0.30 keV given for the 27-keV resonance corresponds to reduced widths of $\gamma_1^2=67.0$ keV and $\gamma_2^2=34.7$ MeV for p- and d-wave neutron interactions, respectively, compared with a single-particle reduced width[17] of $\hbar^2/(\mu R^2)=2.77$ MeV, where μ is the reduced

TABLE II. A comparison of the experimental peak-height data for some narrow levels of F^{20} with the values after correction for the neutron scattering by the tantalum backing for lithium targets. The quantity $\bar{\sigma}$ is the observed cross section and σ_1 the corrected value for a background fraction I_2/I_0, where I_0 is the unfiltered neutron flux and I_2 the flux scattered into the counters by the tantalum. The quantity n refers to the thickness (atoms per cm²) of the transmission sample. The value of σ_1 given in parentheses for each level is the observed value of the peak height to be expected according to the method of analysis given by Monahan and Hibdon. Note that E_r for the first peak is 27.35 keV. The peak heights quoted were the largest observed but are at an energy slightly in excess of E_r.

	Flat-detection				Self-detection			
E_r (keV)	$\bar{\sigma}$ (b)	n (10^{24} atoms/cm²)	I_2/I_0 (%)	σ_1 (b)	$\bar{\sigma}$ (b)	n (10^{24} atoms/cm²)	I_2/I_0 (%)	σ_1 (b)
27.35	64.7	0.0151	4.9	70.1(78.0)	88.8	0.0076	2.9	92.1(90.4)
143.0	5.70	0.0779	3.5	5.77(5.74)	6.15	0.0779	1.4	6.18(5.95)
215.5	6.00	0.0779	6.3	6.13(6.20)	6.55	0.0779	3.75	6.65(6.39)
236.75	7.60	0.0457	6.8	7.86(8.11)	8.65	0.0457	4.4	8.87(8.98)
270.5	14.7	0.020	6.7	15.50(15.88)	15.25	0.020	4.4	15.90(16.06)

[17] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956), footnote 9. See also T. Teichmann and E. P. Wigner, *ibid.* **87**, 123 (1952); A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, 16 (1953).

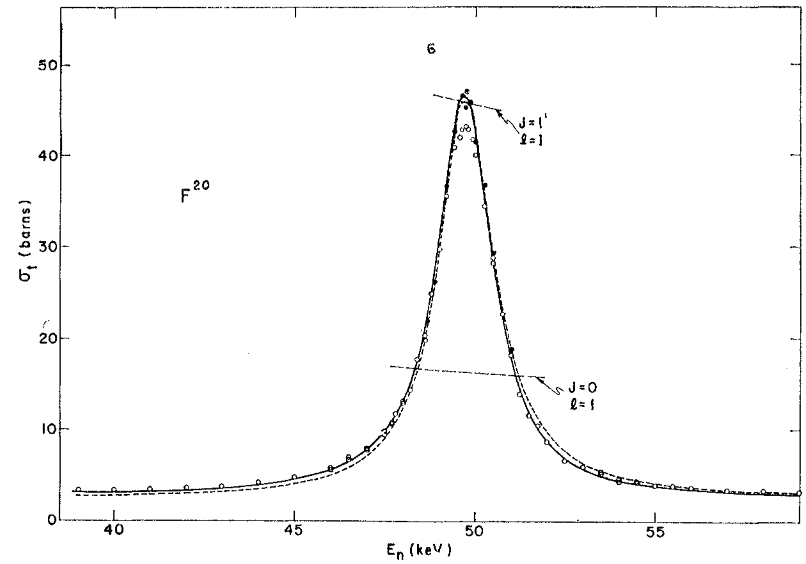

FIG. 8. Neutron cross section of fluorine in the region of the 49.7-keV resonance. Self-detection data are shown by solid circles; flat-detection data by open circles. The solid curve was obtained by a multiple-level plot for peaks Nos. 6 and 9. The broken curve is a single-level plot obtained with the same parameters as were used for No. 6 in the multiple-level plot.

mass. Because the penetrability factor P_l in $\Gamma_n = 2P_l\gamma_l^2$ depends on the value of the nuclear radius chosen, the highly unlikely value of 3.4 b (the average cross section below 15 keV) for σ_p would increase the nuclear radius to 1.37 times the value used and hence reduce values of the γ_l^2 and also would decrease the single-particle reduced width to 1.48 MeV. The width $\Gamma_n = 0.30$ keV would then correspond to $\gamma_1^2 = 26.5$ keV and $\gamma_2^2 = 7.2$ MeV, respectively, for *p*- and *d*-wave neutron interactions. If the *p*-wave, *d*-wave, and single-particle reduced widths are taken to be meaningful, either value of the nuclear radius precludes a value of $l=2$ for this level because the reduced width for $l=2$ exceeds the single-particle width by a sizable factor. If other nearby levels were assumed to be *d* wave, the combination would yield an impossible value for the ratio of the sum of the *d* wave reduced widths to the single-particle reduced width. A value of $J=3$ for the 27-keV resonance corresponds to a minimum value of $l=2$. The corresponding width would then be about 0.20 keV and would yield a value of $\gamma_2^2 = 23$ MeV, which is about 8.3 times the single-particle width for the nuclear radius used here and 3.3 times it for the largest possible radius. Therefore, the 27-keV resonance is attributed to a *p*-wave neutron interaction. It is located at a low energy where an *s*-wave neutron interaction might be expected but no pronounced dip occurs on its low-energy side to identify an *s*-wave interaction. Moreover, a spin of $J=2$ cannot result from the latter interaction.

D. Analysis of the 49.7-keV Resonance

The data in the region of this resonance, shown on an expanded scale in Fig. 8, were obtained with a lithium target having a stopping power slightly less than 0.7 keV. The entrance slit of the collimator to the neutron counter was $\frac{1}{4}$ in. wide. By flat detection a peak height of about 43 b was obtained and by self-detection 45.4 b, compared with a possible single-level height of 45.9 b for $J=1$, $l=1$. A transmission sample of LiF 100 mils thick was used near the peak of this resonance. The 300-mil Teflon detector sample used yields a counting rate equal to 72% of the rate obtained with the 6-in. graphite detector sample. This indicates that the neutron interaction is mainly scattering and agrees with measurements of $F^{19}(n,\gamma)F^{20}$ by Gabbard *et al.*[12] who obtained a peak cross section of 36 mb after correction for resolution. Hence, Γ_n is taken to be nearly identical with Γ because no inelastic neutron scattering has been observed in this region.[13]

This peak is noticeably asymmetric, with the high cross section in its low-energy wing. This type of asymmetry is the reverse of that in an *s*-wave level or for mutual interference with a level at lower energy; but it can result from mutual interference with another level at higher energy. The shape of this peak is generally that of a *p*-wave level, and its apparent width of about 1.75 keV appears to be too large for a *d*-wave level. Hence a value of $l=1$ is assumed. On the high-energy side of peak No. 6, the first level that can interfere coherently is No. 9. This is rather far removed but is quite wide and, as indicated later, is a *p*-wave level. Peak Nos. 6 and 9 are, therefore, analyzed together as *p*-wave levels with $J=1$. The multiple-level plot found to provide the best fit, shown by the solid line in Fig. 8, includes the multiple-level plot of the *s*-wave levels and a small contribution from the multiple-level plot of Nos. 2 and 10 as well as Nos. 8, 12, and 13. Neutron widths of 1.7 and 5.5 keV were used for Nos. 6 and 9, respectively. The curve shown by a broken line is the corresponding single-level plot. It might appear that the single-level plot would coincide with the data if shifted to lower energy. However, a shift of 0.25 keV is necessary to fit the data near 52 keV. For

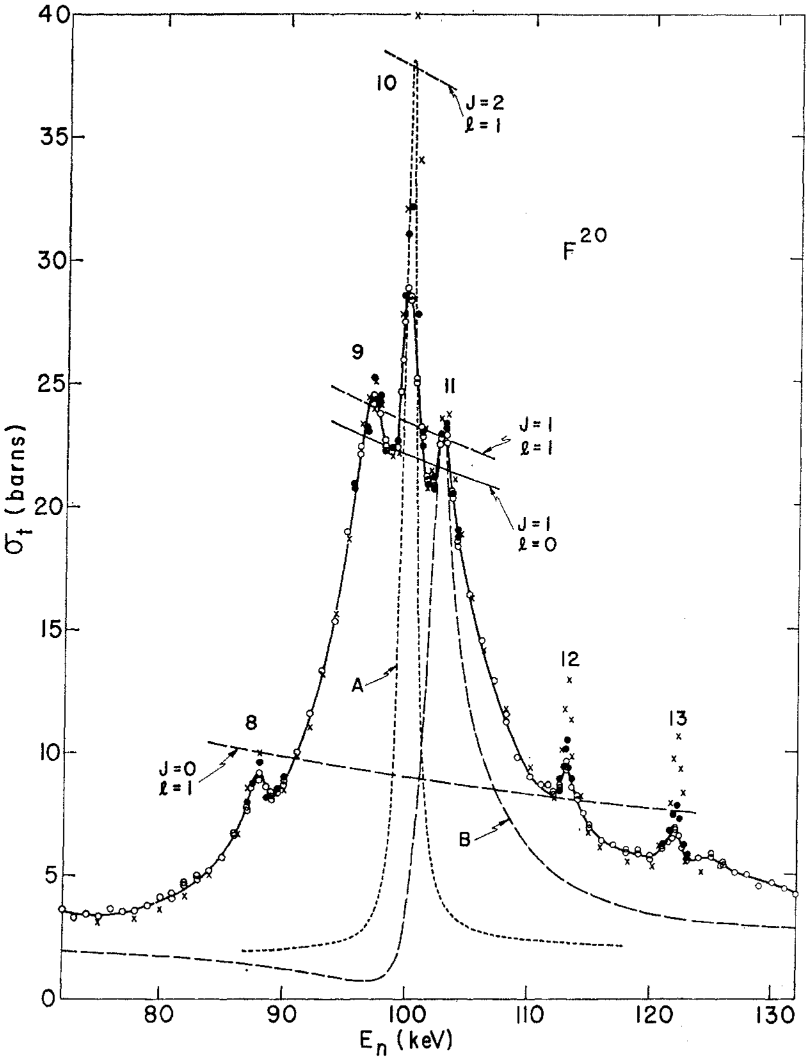

FIG. 9. Neutron total cross section of fluorine in the region near 100 keV. Self-detection data are shown by solid circles; flat-detection data by open circles. Curve A is a theoretical multiple-level plot for peaks Nos. 2 and 10, and a bound level at $E_n=-4.0$ keV. Curve B is a theoretical multiple-level plot of the s-wave levels described in the text. Points shown by crosses represent the sum of all the theoretical curves that extend into this region.

a shift of this amount, the peak of the single-level plot misses the peak of the data by nearly 0.25 keV. Moreover, the single-level plot would fail to coincide with the data in the upper part of either wing of the resonance and would still fall below the data in the low-energy wing.

Data were obtained by both flat- and self-detection in the region from 48.6 to 51.0 keV. These data were corrected for neutron scattering by the tantalum backing for the lithium target[6] and are listed in Table I in the σ column. These data were then analyzed by the method of Monahan and Hibdon.[16] The parameters of this level are taken to be $\Gamma_n=1.60$ keV, $E_r=49.7$ keV, $J=1$, and $l=1$. These parameters were then used to calculate the "expected measurements" listed in the σ_c column of Table I. For multiple-level analysis, a width of $\Gamma_n=1.7$ keV provides a better fit than does 1.6 keV.

The width $\Gamma_n=1.60$ keV corresponds to reduced widths $\gamma_1^2=148.1$ keV and $\gamma_2^2=41.6$ MeV for p- and d-wave neutron interactions, respectively, compared with the single-particle width[17] $\hbar^2/(\mu R^2)=2.77$ MeV for the nuclear radius used. The largest possible nuclear radius previously discussed corresponds to the reduced widths $\gamma_1^2=59.1$ keV and $\gamma_2^2=8.7$ MeV, compared with the single-particle width of 1.48 MeV. The large ratio of the single-particle reduced width to the reduced width for $l=2$ seems to be ample to assign this peak to a p-wave level.

E. Analyses of the Resonance Levels in the 100-keV Region

Although the level structure in this region can be seen in Fig. 1, it is shown more clearly on an expanded scale in Fig. 9. Initial measurements over this region were made with a lithium target having a stopping power of about 0.75 keV. Later the measurements were repeated with a lithium target which had a stopping power

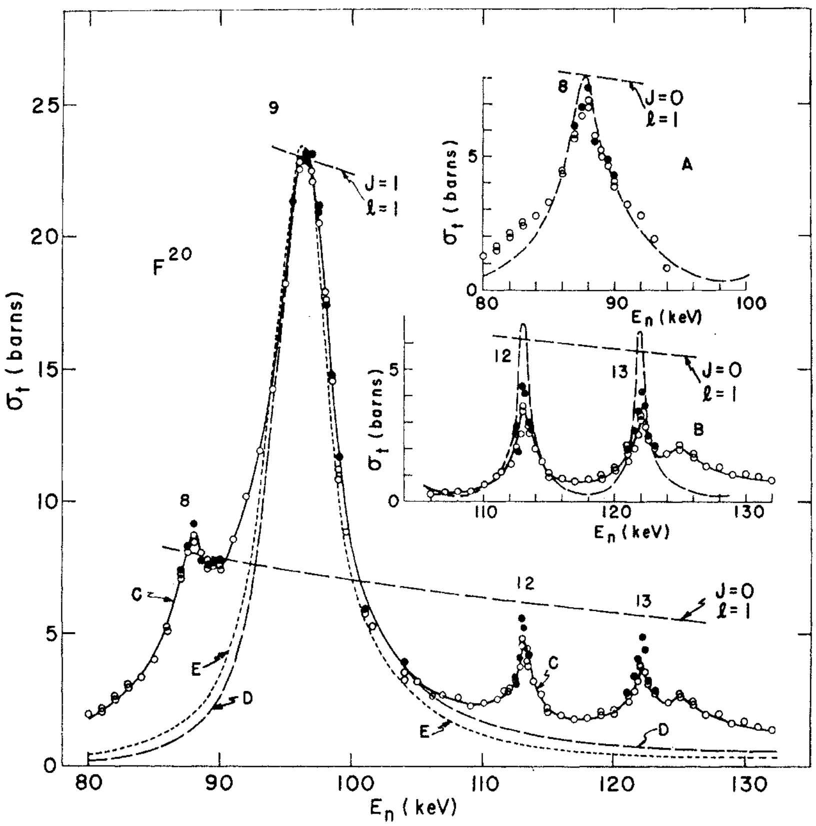

FIG. 10. Analyses of the 96-keV p-wave level and the smaller peaks in its wings. Curve C, shown by a solid line, was obtained by subtractions in Fig. 9 to remove peaks Nos. 10 and 11, the wings of the s-wave levels, and the potential scattering. Curve D is a theoretical multiple-level plot for Nos. 6 and 9; curve E is the corresponding single-level plot for No. 9. The peaks shown in inserts A and B were obtained by subtracting curve D from C. The curve shown by a broken line in each of these two inserts is a multiple-level plot for peaks Nos. 8, 12, and 13.

slightly under 0.70 keV. A slit $\frac{1}{4}$ in. wide was used in the entrance to the collimator of the neutron counter. In the region of peak No. 10, a 100-mil LiF transmission sample was used for both flat- and self-detection measurements. Near the peak of this resonance, the Teflon scattering sample used for self-detection was 125 mils thick; at other energies, it was 300 mils thick.

Several observations indicate that one of the large peaks Nos. 9–11 is due to an s-wave level and that, in fact it is No. 11. (1) There is a large cross section in the high-energy wing of the group. (2) The cross section on the low-energy side of the group is low, particularly from 60 to 70 keV. (3) The fairly wide peak No. 8 is depressed below its single-level height. (4) The low cross section around 60–70 keV is relatively far removed from the group and only a wide level would be expected to depress this region so strongly. (5) The large spacing between peaks Nos. 8 and 9 is more than ample to reveal a deep minimum if peak No. 9 were assignable to an s-wave level. Moreover, the low-energy wing of No. 9 would show a steep rise from the minimum if it were an s-wave level. (6) It appears that the height of peak No. 10, if fully resolved, would correspond to a value of $J \geqslant 2$, which exceeds the value for an s-wave level. Therefore, peak No. 9 is taken to be a p-wave level and No. 11 an s-wave level, both with $J=1$. Peak No. 10 is then due to a narrow p- or d-wave level. The remainder of the peaks shown in Fig. 9 appear to be attributable to $J=0$.

Angular-distribution measurements in this region made by Lane[18] some time ago, indicated strong interference between s- and p-wave levels and a preference for low values of J. No individual levels could be resolved because of the large neutron-energy spread. Spin and parity assignments proposed in the present case are therefore not inconsistent with the angular-distribution measurements in that the group appears to contain one relatively wide s-wave level, a wide p-wave level, and other p-wave levels. No peak height appears to correspond to $J>1$ except peak No. 10.

The only method of untangling this cluster of levels appears to be trial and error. Therefore, a set of provisional parameters was used to obtain the multiple-level plots. These plots were compared with the data and the parameters were varied until the plots provided the best obtainable agreement with the data. The wings of all known levels that extend into this region (except peak No. 7) are included in the plots. The sum of the wings of all the levels are shown in Fig. 9, where the points are marked by crosses. These points seem to agree with the data except at the peaks of the narrow levels that could not be completely resolved to their

[18] R. O. Lane (private communication).

TABLE III. Levels of F^{20} derived from neutron reactions with F^{19}. The parameters J, l, and Γ are the values obtained as a best fit to the data. The parameters for the s-wave levels used in the analyses are given also.

No.	E_r (keV)	J	l	Γ (keV)
1	15.25			
2	27.35	2	1	0.30
3	31.9			
4	34.0			
5	38.0			
6	49.7	1	1	1.70
7	70.5			
8	87.8	0	1	4.00
9	96.0	1	1	5.50
10	100.0	2	1	1.10
11	102.4	1	0	3.00
12	113.2	0	1	1.20
13	121.7	0	1	1.00
30	200.2	0	0	2.0
43	246.0	1	0	1.8
44	247.8	1	0	0.4
48	267.7	1	0	1.4

true peak heights. The high-energy wing of peak No. 7 could not be included because this level was so narrow that its parameters could not be inferred from the present data.

It is necessary to know to what extent the wings of any particular level are distorted by the wings of other levels. Therefore, a plot is shown for each level and these individual contributions are successively subtracted from the experimental curve. First, the plots for peaks Nos. 10 and 11 are shown in Fig. 9. To avoid confusion, only two peaks are plotted in this figure. Curve (A) for peak No. 10, obtained with a neutron width of 1.1 keV, is a part of the p-wave multiple-level plot for Nos. 2 and 10 and a bound level at -4.0 keV. Curve (B) is the multiple-level plot of the s-wave levels, calculated for a width of 3.0 keV for No. 11. The parameters of the other levels were those given in Sec. 4B. Each of the two curves (A) and (B) includes the potential scattering. Hence, by subtracting curve (B) and the resonance part of curve (A) (i.e., the contributions of peak No. 10 exclusive of potential scattering) from the data, one obtains the resonance contributions from the remaining peaks, shown in Fig. 10 by curve (C). Therefore, the potential scattering is not included in any curve in Fig. 10.

Peak No. 9, if a single level as it appears to be, is a wide level. It exhibits the general shape of a p-wave level and appears to be resolved to its true peak height. The multiple-level plot for peaks Nos. 6 and 9, shown by curve (D), was obtained with a width of 5.5 keV for No. 9. The corresponding single-level plot is shown as curve (E). By subtracting the wings of curve (D) from curve (C), one obtains the points for peaks Nos. 8, 12, and 13 shown in inserts A and B of Fig. 10. The small peaks Nos. 12 and 13 were corrected for the neutron scattering by the tantalum backing of the lithium target[6] (a very small correction) and then analyzed by the method developed by Monahan and Hibdon.[16] This method indicates that these two peaks, if fully resolved, should attain peak heights of about 6.6 and 6.3 b, respectively. These peak values are close to the single-level peak heights for $J=0$. The broken lines in inserts A and B are the multiple-level plots for peaks Nos. 8, 12, and 13, obtained with the parameters listed in Table III.

Next, the wings of levels 8, 9, 11, 12, and 13 were subtracted from curve (C) in order to obtain the points shown in Fig. 11. The successive subtractions can be expected to introduce sizable uncertainties in the points shown in Fig. 11, but these points nevertheless outline a reasonable resonance shape. The points from 99 to 101 keV were corrected for the neutron scattering by the tantalum backing of the lithium target.[6] This correction increases the peak height a little more than 1 b. Then the method of analysis developed by Monahan and Hibdon[16] indicated a width of 1.1 keV and a fully resolved peak height of about 34 b, compared with a single-level height of 37.7 b for $J=2$. The computed values for a 0.4-keV neutron-energy distribution are shown in Fig. 11 by a solid curve for flat detection and a broken curve for self-detection.

Peak Nos. 10, 12, and 13 were assumed to be due to p-wave levels but a d-wave neutron interaction is not necessarily ruled out. For peak No. 10, the neutron width of 1.1 keV corresponds to reduced widths of $\gamma_1^2=37.4$ keV and $\gamma_2^2=5.2$ MeV, respectively, for p- and d-wave neutron interactions. This is to be compared with the single-particle reduced width[17,19] of 2.77 MeV. For No. 12, the assigned width of 1.2 keV corresponds to reduced widths of $\gamma_1^2=33.7$ keV and $\gamma_2^2=4.1$ MeV; and for No. 13, $\gamma_1^2=25.2$ keV and $\gamma_2^2=2.8$ MeV. Hence, for $l=2$, the reduced width of any one of these peaks is comparable to or exceeds the single-particle reduced width and, for all three, the combined reduced width is about 5 times the single-particle width. Moreover, the widths of the small peaks shown in Fig. 3 appear to be comparable to the widths of Nos. 12 and 13. Since these peaks occur in an energy region much narrower than the width of a single-particle peak, the sum of the reduced widths of these peaks should be considerably less than the single-particle reduced width for the correct value of l for the levels. For $l=1$, the sum is indeed much less, but for $l=2$ the sum exceeds the single-particle width by a large factor. Tentative values of the widths of the peaks in Fig. 3 were used.

V. RESONANCES IN THE 130–300-keV REGION

Figure 3 shows a region of many small peaks. A correction[6] for neutron scattering by the tantalum backing does not increase the heights of these small

[19] A. M. Lane, R. G. Thomas, and E. P. Wigner, Phys. Rev. 98, 693 (1955).

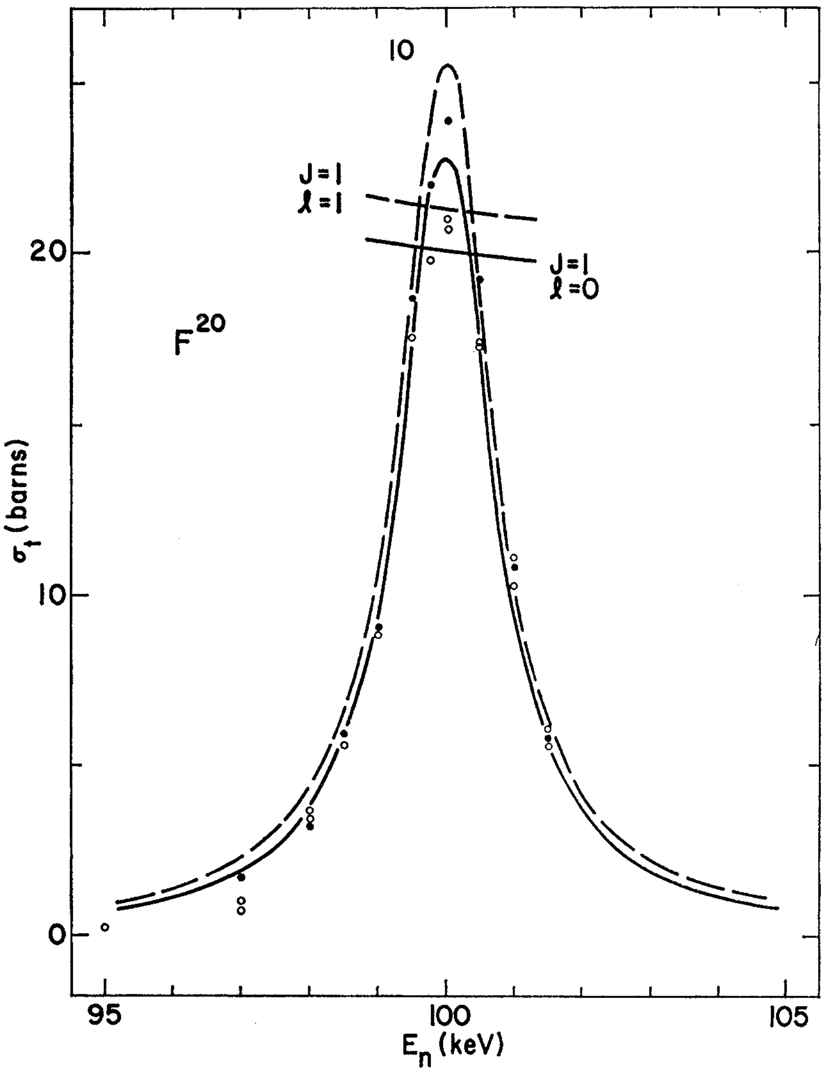

FIG. 11. The plot for peak No. 10, obtained by subtracting the wings of all other resonances in this region from the data in Fig. 9. The solid curve represents the flat-detection data to be expected, as obtained by the method of analysis given by Monahan and Hibdon. The dashed curve shows the expected self-detection data.

peaks appreciably, as is shown in Table II. An attempt was made to analyze these peaks by the method of Monahan and Hibdon.[16] However, the difference between the theoretical peak heights for successive values of J is quite large, and the peak heights obtained by the analyses do not come very close to particular values for any J. Rather, they fall more or less midway between the values for different J. Inelastic scattering is known to begin in this region[13] and is apparently strong enough to reduce the peak heights considerably. Hence, $\Gamma_n < \Gamma$ and, consequently, the peak heights are reduced by the factor Γ_n/Γ. Although larger peak heights were observed in the region covered by Figs. 4 and 5 than for Fig. 3, the analyses are still uncertain. Tentative sets of parameters have been obtained for the levels shown in Figs. 3–5, but it is hoped that a better analysis can be made when a method is devised to include a reasonable estimate of the inelastic scattering.

The parameters obtained for the levels that were analyzed are collected in Table III. These parameters appear to account for the levels very well. Below 15 keV, the cross section could not be explained unambiguously. The assumed bound levels that were used contribute approximately the correct amount, but these levels may only be fictitious and equivalent to the combined effects of many levels. However, the true value of σ_p appears to be less than 3.4 b, so that some combination of bound levels apparently contributes to elevate the cross section in the low-energy region.

VI. STRENGTH FUNCTIONS

The resonances that contribute appreciably to the strength functions were analyzed up to about 130 keV and the average strength function can be evaluated for these resonances. It is given by the expression[17,19,20]

$$\text{S.F.} = \langle \gamma_l^2 \rangle_{\text{av}} / D_l , \tag{1}$$

[20] H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev. **96**, 448 (1954).

the ratio of the average reduced neutron width to the average level spacing, where the reduced width $\gamma_l^2=\Gamma_n/(2P_l)$ and the average level spacing D_l are evaluated for a given value of l. Only five unbound s-wave levels were identified up to 300 keV, so the strength function averaged for both possible spins is 0.04. This value is small, as predicted by the optical model.[20] Averaged over all three possible spins for p-wave levels, the strength function is 1.8. The small peaks Nos. 1, 3, 4, 5, and 7 shown in Fig. 1 could not be included because their widths are too small to be evaluated. However, the ratio given by Eq. (1) is equivalent to $\sum_r \gamma_r^2/\Delta E$, where ΔE is the energy interval covered. If these small peaks are assumed to be p-wave levels with an average width of 10 eV, the strength function would be increased to a value not in excess of 1.9. At any rate, their inclusion would serve to further increase the very large strength function already obtained.

The value of P_l, and hence γ_l^2 and in turn the strength function, depends on the nuclear radius, which cannot be unambiguously determined from the present data. An upper limit of 3.4 b for σ_p comes from the cross section below 15 keV in Fig. 1. This unlikely value of the potential scattering would require a nuclear radius some 37% larger than the one used. Use of this high value would lead to a p-wave strength function of about 0.9. Hence, for either value of the nuclear radius, a large value of the p-wave strength function is obtained. This is in accord with the optical model,[20] to the extent that this model predicts a maximum in the p-wave strength function in the region $A=15$–30.

The ratio $\theta_l^2=\gamma_l^2/(\hbar^2/\mu R^2)$, the fraction of the single-particle width of a level,[21] is usually regarded as a measure of the degree to which a level arises from the excitation of a single nucleon. The numerical value of θ_l^2 should then be about 1 for single-particle levels and much less for more complex excitation processes. In an energy interval of magnitude comparable to the width of single-particle levels, the sum of the fraction θ_l^2 for all levels for a particular l should also be approximately 1. For the levels of F^{20} that could be analyzed and could be attributed to p-wave interactions, the sum $\sum_r \theta_l^2$ over all resonances r was found to have a value of 0.25 times the single-particle limit $\hbar^2/(\mu R^2)$—or about 0.20 times this limit if the largest possible value of the nuclear radius is used. As mentioned earlier, an attempt was made to analyze the resonances in the region 130–300 keV and tentative values of the Γ_n were obtained. However, the effects of inelastic scattering were ignored. Hence the widths obtained are believed to be too large. Even so, if all of the levels that could be attributed to p-wave interactions are included, the sum $\sum_r \theta_1^2$ for all levels up to 300 keV is about 0.42 times the single-particle limit. Hence, for all of the observed peaks in F^{20} either up to 130 keV or up to 300 keV, the sum $\sum_r \theta_1^2$ appears to be quite large for so small an energy interval. This large fraction is interpreted to mean that F^{20} is located close to the peak of the p-wave maximum of the giant resonance.

[21] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. 30, 257 (1958).

VII. DISCUSSION

Considerable effort was devoted to a determination of the potential scattering σ_p because some value of this quantity is necessary to fit the wings of levels as a basis for interpreting the data. However, no ambiguous value of σ_p could be obtained from the data. The wings of the levels that could be analyzed could be reasonably well accounted for by taking $\delta_0=R/\lambda\!\!\!^-$ for the phase shift associated with the s-wave potential, calculating the potential scattering from this, and adding the combined wings of all levels (including some assumed bound levels) that were known to contribute in this region. This method yields both the background on which a level sits and the effect of mutual interference with other levels. An alternative method which yields equivalent results is to add a second term to $R/\lambda\!\!\!^-$ to account for the wings of other levels. Because sizable variations may occur from level to level, particularly for s-wave levels, the second term in δ_0 may be about as uncertain as the combined wings of the levels themselves. Moreover, the minima of s-wave levels can be accounted for by the method used in this paper, but the deep minima observed with these levels in some cases may be difficult to explain by use of a second term in δ_0. The procedure used in this paper aided in obtaining fits to the levels far out in their wings. Moreover, the method used here includes the mutual interference with relatively far removed levels in that actual multiple-level plots were used. The results obtained in this way were combined with the wings of noninterfering levels.

The results of the analyses obtained by the method given by Monahan and Hibdon[16] provided fairly definite values of the widths, locations, peak heights, and hence the spins of the levels which do not occur in groups. The parameters were used to obtain single-level plots over the region of a level, and these plots were compared with the data. In this way the failure of a single-level plot to coincide with the observed shapes of the wings of a level provided a strong indication of mutual interference with some other levels. The type of level and the amount which its peak height is depressed or increased by mutual interference were taken into account in assigning the spin and parity. It is felt that the method used leads to more reliable values of the spin and parity than a mere visual comparison of the observed peak heights with possible theoretical peak heights, although these values are not necessarily unique. The spins and parities assigned to peaks Nos. 2 and 6 were the same as those assigned several years ago here. However, the present data indicate that peak No. 6 is nearly completely resolved to its true peak height—so nearly that the peak height computed by the method of Monahan and Hibdon[16] was no greater than was directly obtained by self-detection or flat-detection. Moreover, the present data appear to rule out mutual interference between peaks Nos. 2 and 6. Work at Duke University[5] led to these same spin and parity assignments for these two peaks. The latter assignments were made on the basis of transmission and angular-distribution measurements. It thus appears that the parameters of these two levels are determined quite well.

An analysis of the group of resonances near 100 keV was based on the absence of other levels wide enough to interfere with those analyzed. In order to detect as many as possible of the narrow levels, data for both flat-detection and self-detection were obtained at closely spaced intervals. It was hoped that where the resolution was insufficient to detect a level by flat-detection its presence would be indicated by self-detection. Total cross-section data[5] obtained at Duke University also show irregularities in the region near peak No. 8 of the present data, but the intervals between points seem to be too large to resolve the remaining structure of this group of levels.

The many small peaks shown in Figs. 3–5 were first observed by flat detection. Later, the entire group was restudied by self-detection and the peaks were observed again. Still later measurements by both flat-detection and self-detection at each energy setting over the entire region are shown in Figs. 3–5. The large number of small peaks observed raises the question that some of these peaks may be instrumental in origin. However, for every experiment, all equipment is carefully aligned. A complete set of in-out transmission measurements was made at each energy setting. This could be done fairly rapidly for a given level. The counting rates were carefully checked and do not appear to be responsible for these peaks.

Despite misgivings about the large level density above 130 keV some other data also seem to indicate the presence of a number of peaks. Freeman,[13] who used neutron energy spreads of 30–40 keV, observed an unusually large inelastic-scattering cross section in this region. Her data may be interpreted as due either to the presence of many narrow levels or a few wide levels. However, if only a few wide levels occur in this region, the inelastic cross section should have revealed their presence. Hausman *et al.*[22] found relatively narrow levels in sodium by inelastic-scattering measurements with fairly large neutron-energy spreads. Their yield curve shows some large peaks where groups of levels were later found to exist. In the region of 500–600 keV, it also shows narrow peaks that were later found to have widths less than 5 keV.[1]

Polarization data obtained by Elwyn *et al.*[23] in the region of these small peaks show little change with a variation in neutron-energy spread from about 30 to 50 keV. Their results may then be interpreted as due to the presence of many narrow levels. On the other hand, it may be interpreted as arising from the slow variations in the wings of wide levels. However, the results given in the present paper and the results obtained at Duke University[5] do not show wide levels in this region. The polarization data obtained by Elwyn *et al.*[23] also indicate the presence of s- and p-wave interference in the region near 100 keV.

It is also noteworthy that the p-wave strength function up to about 130 keV appears to be quite large whether or not one considers the structure near 100 keV as a single wide level or a group of narrow levels. This indicates that F^{20} occurs near a giant p-wave maximum. Since the region of energy covered by a giant resonance is known to be large, a large strength function should obtain over the region of the giant maximum. However, the p-wave strength function of F^{20} will show an abrupt drop in the region 130–260 keV, where no wide levels have been observed, unless a sizable number of narrow levels exist there.

The discussion concerning level density in the region 130–300 keV is merely an assemblage of the pertinent information and is not intended to confirm or deny the observed level density. A number of levels appear to exist in this region but a one-to-one correspondence of the observed peaks with levels of the compound nucleus is not certain at present. Conceivably one or more of the following factors could introduce narrow peaks which do not correspond to levels of the compound nucleus: (1) undetected impurities in the samples, (2) instability of the accelerator, and (3) the fact that the accelerator is operated to produce narrow neutron energy spreads near its limiting value. On the other hand, it should be noted that small peaks did not occur in the data on lithium (Figs. 1 and 2).

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[22] N. Hausman, J. E. Monahan, F. P. Mooring, and S. Raboy, Bull. Am. Phys. Soc. **1**, 56 (1956). The curve for their results is shown in the following reference: R. O. Lane and J. E. Monahan, Phys. Rev. **118**, 533 (1960).

[23] A. J. Elwyn, R. O. Lane, and A. Langsdorf, Bull. Am. Phys. Soc. **8**, 513 (1963).